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Focus on Middle and High School Issues

A Naturalistic Look at Language Factors in Mathematics Teaching in Bilingual Classrooms₁

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Introduction

Until recently, it has been assumed that language factors did not play a significant role in our understanding of the teaching and learning of mathematics. Classroom discourse characteristics had been studied as part of a general focus on research on teaching (Cazden, 1986) or as part of a way of understanding the learning of such areas as language arts, but not mathematics. The belief that mathematics transcends discourse factors has been pervasive and even has affected bilingual education policies in that mathematics often is promoted as one of the first subjects that can be taught in the students' weaker language although other areas continue to be taught in the students' native language.

But why study language factors in the teaching of mathematics, and particularly in bilingual classrooms? The present study begins to answer this question with an investigation of how language is used by teachers to introduce new mathematical concepts to limited English proficient (LEP) and non-English proficient (NEP) students. The study reflects the assumption that the learning of mathematics is based on the essential ingredients of people engaged in communication for the purpose of developing shared meanings and understandings. Within this perspective, little is known of how teachers use language to explain mathematics as part of the conceptual development process, and thus, how ability is affected by these explanations and other discourse characteristics. This gap in our knowledge is particularly evident when it comes to multilingual classroom contexts.

The research reported here is part of a larger qualitative study of mathematics teaching with students of Mexican heritage. The discussion that follows is specifically directed toward issues related to communication factors in early middle grade classes where rational number concepts are developed. Furthermore, whereas the study has focused on issues involving Spanish-speaking students, the concepts and insights that emerge apply equally well to other language groups.

Conceptual Framework

Our discussion should begin with a brief look at the patterns of mathematics achievement among students of Mexican heritage. One such pattern was noted by Matthews and her colleagues (Matthews, Carpenter, Lindquist, and Silver, 1984) as they compared different year's national assessment results. Recent results indicated that there had been small gains in computation and no gains in problem solving and application. A second pattern is that the gap between some Hispanic groups and majority students widens in the areas of knowledge and skills, mathematics understanding, and application as students get older (National Center for Education Statistics, 1990). Interestingly, the gaps do not significantly appear until the middle elementary

grades and continue to increase thereafter.

Although there has not been extensive analysis of specific test items to suggest possible factors that might explain why performance should be significantly lower in the areas of problem solving/understanding and application, we can make some assumptions about the nature of these items. The questions in these sections likely would be more word laden thus requiring the ability to interpret and translate into mathematical symbols. The questions clearly would also require a stronger conceptual foundation than straightforward computation. In both cases, language would play important roles: first, in the comprehension of questions, and second, at the earlier point when conceptual understanding was being developed. It is significant that the middle grades appear to be a turning point because it is also here where the traditional emphasis in mathematics learning shifts from simple whole number facts to more conceptually complex kinds of numbers, mathematics, and application.

Research to examine the relationship between language and mathematics achievement generally has focused on simply comparing reading and mathematics scores (e.g. NAEP, 1980). Other studies have examined difficulties and confusions that can result from the unique ways problems are translated into mathematical symbols (e.g., Mestre, 1981) and from the differences in ways mathematical concepts are expressed in Spanish and English (Cuevas, 1984). However, the issue has not been studied from an ethnographical perspective to discern not only how language and mathematics interact but also how teachers explain mathematics in a multilingual context.

As noted earlier, a major premise of this study is that the learning process is highly embedded in communication. Within this framework, the teacher is not simply the person who speaks in isolation or who doles out information to students. Rather, the teacher is the person who initiates, structures, and develops shared meanings with students. Vygotsky (1978) suggested that learning and higher psychological processes are the result of meaningful interactive social experiences. Such experiences are generally found early on in the highly communicative relationships between parents and young children. Vygotsky (1978), having observed these interactions, placed great importance on the role of the parent as an "enabling other" in the learning process, and suggested that the interactions in this process were similar to those that occur in an apprenticeship. The enabling other, therefore, is the more experienced person who embodies and models the intended learning outcomes. Consequently, it is this person who also enculturates or draws the novice into intended common experiences, meanings, and modes of thinking.

Previous work on teacher discourse in instruction has noted the prevalent tendency for teachers to use a scripted type of talk where the teacher asks a question, the student responds, and the teacher acknowledges the response (e.g., Mehan, 1979). However, this is not the type of language that can be used for conceptual understanding. The language that is used in explanations, like that which a mentor uses with an apprentice, is less automatized and evaluative. Instead, explanations "...develop an awareness of what is being taught, when it will be used, how to do it, and how it is different and/or the same from previous concepts" (Duffy, Roehler, Meloth, and Vavrus, 1986, p. 204). The teacher must also "select and construct models, examples, stories, illustrations, and problems that can foster students' mathematical development" (Ball, 1990, p. 3). However, as Duffy and his colleagues (1986) point out, little is known about how teachers actually explain concepts to students and what effect this has on student cognitive processing. This is particularly true in mathematics even though it is recognized that effective teachers include a substantial development portion in a daily lesson (approximately twenty minutes) and this portion is characterized by very clear and unambiguous teacher talk (Good, Grouws, and Ebmeier, 1983).

The Language of Mathematics

Mathematics is a different and difficult subject to explain partly because of the language we use to communicate mathematical ideas and also because the ideas are not straightforward. To fully understand the problems NEP and LEP students might have in learning mathematics, let us look at some of the issues involved in thinking and speaking mathematically.

Up to fifth grade, the traditional mathematics curriculum is concerned with concepts that may be described as having to do with calculating gains, losses, comparisons (e.g., How much older is Jose than you?), and shares (e.g., If there are a hundred oranges and five boxes, how many oranges will go into each box?). Although some of these concepts can present problems to students when they are expressed in word problems (e.g., Juanito has twice as many apples as you, and you have four apples. How many apples does Juanito have?) (Orr, 1987), students, nevertheless, are using only whole numbers. As students reach fifth grade, they are presented with the need for numbers beyond those to which they are accustomed.

Arithmetically, the fraction $\frac{2}{3}$ becomes necessary as the only solution to the whole number problem 2 divided by 3....and we need more precision to measure than to the nearest inch. (National Council of Teachers of Mathematics, 1989, p. 91).

Students also now must deal with a complicated array of meanings that only partially overlap (Ohlsson, 1988). For example, a *half* of a piece of paper is only a *half* when it is placed in relationship to another piece of paper that has been defined as the original whole; a half of a piece of paper can just as easily be thought of as a whole piece when there is no larger referent. Three-fourths of an apple is not exactly the same as $\frac{3}{4}$ of a melon although the fractional parts have the same relationship to their respective wholes. The point is that students are now confronted with comprehending and identifying units and part/whole relationships that they did not have to do before. Moreover, fractions can have a breadth of meanings beyond part/whole terms. They can be interpreted (1) as a number on the number line; 2) as an operator that can shrink or stretch another quantity (e.g., $\frac{1}{2} \times 12 = 6$); (3) as a quotient of two integers (e.g., 2 divided by 3); and (4) as a ratio (e.g., 3 out of 4 people...) (Ball, 1990; Behr and Post, 1988).

In addition to being presented with a new type of number and new meanings, students must learn new ways of expressing these concepts, or a new register. Halliday (1978) used "register" to refer to not only special terminology, but also the use of natural language in a way that is particular to a role or function. Everyday words become reinterpreted as part of a set of unique meanings and structures. For example, "right", "point," and "left" have very different meanings outside of the mathematics context—and often different meanings from what students expect when they are used in mathematics. For instance, children first learn that "right" means a direction or correctness. However, "right" is used in geometry to refer to an angle with special characteristics that have nothing to do with either of the former definitions. The middle-grade mathematics register includes not only new terms such as "denominator," "numerator," "equivalent fractions," or "perimeter," but also terms that, when mistakenly thought of in the context of the natural language, become incomprehensible because of conflicting meanings. "Lowest common denominator" is one example. Students can interpret "lowest" to mean something suggesting a physical placement of the denominator rather than a "simplified" quantity. If the specialized meaning is taken for granted and not pointed out to students, there is a risk that it could be misunderstood. Other terms can easily be confused because of the way they sound, for example: "holes" for "wholes" and "2 fours" for "2 fourths."

Although the register may cause students difficulty in thoroughly comprehending mathematical discussions,

it should not be avoided. Instead, development of the alternative meanings and multiple terms should be included in the mathematics lessons.

At this same time, the mathematics is expanded to include both arithmetic and geometry. With the introduction of geometry, comes the new concepts of space, distance, and shape, along with additional register terms such as "planes," "faces," "right triangles," and "width" as opposed to "with." Furthermore, as the National Council of Teachers of Mathematics (1989) has noted students must have a deeper sense of what, when, and how to use numbers.

Students are required to know the difference between $14 + 67$ and $1.4 + 6.7$. At one point, we say $2/4 = 1/2$ and then mark incorrect an answer of $2/4$. However, we prefer $68/100$ to $17/25$ dollars on checks... (p. 88).

Students also must recognize that various symbolic forms can represent the same quantity as in $1/4$, $25/100$, 0.25 , and 25% . The importance of these points is that learning mathematics involves much more than has previously been conceived. The mathematics register can be a formidable factor particularly when new and complicated meanings are being developed and when spoken words can easily be confused or interchanged because of unclear pronunciation. As Pimm (1987) has aptly described, the teaching of mathematics, in general, can be characterized by ambiguous word referents and gross misunderstandings of the spoken language.

Bilingual Learners

The present study is concerned with a very unique population of students. Consequently, the last background dimension we should consider relates to the relevant aspects of what is known about effective instruction for LEP and NEP students.

It is well recognized that the use of a child's primary language (L1) has clear beneficial effects on school progress particularly when it is used in the instruction of concepts and for clarification (e.g., Cummins, 1981; Tikunoff, 1983; Wong Fillmore and Valadez, 1985). This is consistent with the premise that students learn best in the language they comprehend best. In fact, Wong-Fillmore and her colleague (1985) report that students were more involved in learning and participated more actively in those classes where L1 was used. With regards to mathematics, specifically, Coffland and Cuevas (1979) found a direct relationship between instruction in the student's first language and high achievement in the subject.

Effective instruction also includes the integration of English language development (L2) with academic skill development (e.g., Tikunoff, 1983). In other words, as a subject is taught, attention is paid to students' second language acquisition of new terms for concepts or simply new vocabulary and syntax modes. In those instances when instruction is in the student's weaker language, careful attention must be given to the speech acts. The speech must have

(1) a slower rate and clearer articulation, which helps acquirers to identify word boundaries more easily, and allows more processing time; (2) more use of high frequency vocabulary, less slang, fewer idioms; and (3) syntactic simplification, shorter sentences (Krashen, 1982, p. 64).

However, it is important to note that positive results of bilingual education are best achieved through the separate use of the primary and second languages. The languages should not be mixed, but rather should be used at different times and for distinct purposes (California State Department of Education, 1984). In essence, LEP and NEP students require what August and Garcia (1988) call "understandable substantive

instruction" along with integrated second language development.

Methods

The present study is based specifically on the observations conducted in two middle grade classrooms. The classrooms are in schools that have a significant population of Hispanic LEP and NEP students and are in the same district. The bilingual program in this district can be considered transitional in keeping with the state's policy. However, the program in this particular district is considered more progressive by other districts in that its bilingual education program is not viewed as "special services"; rather, it is an integrated school program with bilingual teachers found at almost every grade level.

The teachers who are the basis of our discussion are part of a larger project and, as such, had been identified by school administrators and bilingual and mathematics curriculum directors as being teachers who were considered to be effective in the teaching of mathematics with Hispanic students. These teachers did not have a specialization in mathematics nor in a related subject such as science, and had only had mathematics methods courses as part of a traditional teacher preparation program. Interestingly, they both indicated that they enjoyed teaching mathematics and took much pride in the emphasis they gave to teaching the subject.

Both teachers have certification in K-8 with a specialization in bilingual education, have approximately the same number of years experience teaching at the middle grade level (four to five years), have had all of their schooling in the United States, and are native Spanish speakers. One teacher, however, often expressed a concern that his technical vocabulary in Spanish was weaker than he wished since he had never been taught advanced mathematics in Spanish and what relevant vocabulary he knew he had learned on his own. The other teacher felt confident about his command of Spanish for teaching mathematics. At the time of this study, both teachers were teaching at the same grade level although they were teaching different aspects of the curriculum.

The two classrooms were videotaped for seven to ten hours specifically on days when the teacher indicated that new concepts would be explained. The classrooms also were videotaped for one entire week (some of the hours mentioned previously are included in this period) to get a sense of the consistency of the mathematics lessons across time. In addition, one entire day was videotaped in each classroom in order to get a sense of how mathematics related to the rest of the curriculum. Field observations were conducted intermittently in each classroom for one year and various artifacts, such as completed student worksheets, were collected to supplement the videotapes that are the primary source of data.

Each teacher was interviewed with a prepared set of questions regarding personal and professional background, perceptions of teaching mathematics, and academic and language characteristics of his students. Informal interviews were conducted with randomly selected students to assess their grasp of the mathematical meanings presented in the lesson and to enhance the observations.

The analysis of the videotapes focused on selected constructs: (1) the nature and use of a mathematics register; (2) the nature of explanations for concept development; and (3) the nature and use of L1 (Spanish) and L2 (English). Triangulation among three independent observers was used to provide validation of the items deemed to be linguistically troublesome.

Lastly, the Hispanic students in both classrooms represented the full spectrum of proficiencies in Spanish and English; some students spoke only Spanish, some were fully bilingual, and some were English

dominant. In one classroom, students of Mexican heritage comprised approximately two-thirds of the class with the remainder being made up of non-Hispanic students. In the other classroom, only one student was not Hispanic.

Results and Discussion

The Absent Mathematics Register

Analyses of the data present three striking patterns of teacher discourse. The first centers around the little attention teachers gave to the mathematics register. Very few mathematical words or phrases were actually spoken regardless of whether fractions were taught in one classroom and decimals in the other. The teachers would open the day's lesson with a perfunctory naming of the objective, such as "adding like fractions" or "adding decimals" and would offer the Spanish version of these ("sumando fracciones" or "sumando decimales"). Aside from these initial introductory statements and occasional corrections or affirmations of student responses to problems, few mathematical words or sentences were said. The teachers put problems on an overhead machine and students took turns solving the problems. There would be active interchanges between teacher and students; but this talk would contain few mathematical words or incomplete sentences or ambiguous phrases. For example, the teachers often read quantities as a series of single digits as in the case of: "Add one, three, seven, and eighty-two" instead of "Add one hundred and thirty-seven and eighty-two" ($137 + 82$).

The following description of a lesson further illustrates the pattern of limited mathematical speaking. The teacher in this example states at the beginning that this lesson is intended to teach how to subtract fractions with the same denominator. The lesson begins with a general question of "Who can tell me what a denominator is?" Several students, en masse, call out that the denominator is the *number on the bottom* that indicates the physical placement of the number but does not offer any mathematical meaning. The teacher accepts this and then provides an example of the kind of problems the students will soon be solving. The teacher calls attention to the following problem, that is written on the overhead:

$$\frac{2}{3} - \frac{1}{3} = \underline{\quad} .$$

He then demonstrates how to solve it by saying "two minus one is one" and writes in the blank the answer "1/3." The names of the quantities are never used; "two thirds minus one third equals one third" is never stated. Immediately following this, students are shown the set of problems in Figure 1 on a transparency that they solve together as a group.

Figure 1 was not included in the electronic version of this publication.

During this time, the names for the fractions, again, are infrequently verbalized by either teacher or students. In another lesson, the teacher writes: $\frac{3}{4} - \frac{1}{3}$ and simply says, "Okay, now do this one" without naming the fractions or verbalizing the implied question either before or after presenting the problem.

The teachers also appeared to have difficulty using the Spanish mathematics register. Mistakes in naming fractions were quite frequent. In the following dialogue, we can see not only how errors occur but also how a pattern of misspeaking is reinforced between teacher and student.

Teacher (T): Right. Fracciones equivalentes, ¿Verdad? (*Equivalent fractions, right?*) Now, well, I'm gonna

show you right now, let me show you for example see if you can remember.

(The teacher holds up a piece of candy that has sixteen sections.)

Now there are sixteen and I broke it and I cut it in half a little bit. Now, what is an equivalent fraction to a half?

Student (S) 1: Eight sixteens.

T: Eight sixteens. Oh, what's another way of sayin' it? How 'bout another way?

S 2: Three fourths.

T: Three fourths to a half?

S 3: Four eights.

S 2: Four eights, yeah.

T: Now, wait a minute. Hang on. Is this the same thing? Yeah.

(The lesson moves on to a different example.)

In this next example, the other teacher is introducing decimals to his students. He writes the problem $13 + 17$ vertically on the overhead.

T: Hoy vamos a hablar de adición *(Today, we are going to talk about addition)*. Addition is very easy. Es muy facil. *(It's very easy)* Es muy simple. *(It's very simple.)* Let's do this juntos *(together)*.

(The teacher and students work the problem step by step and get a answer.)

We're gonna add a little bit more to this.

(The teacher draws a large dot on the transparency, but not near the problem that has just been solved.)

Who knows what that is *(pointing to the solitary dot)*?

S 1: Un decimo. *(A tenth)*

T: Right, decimo *(tenth)*.

(The teacher and students begin switching back and forth saying decimo for the Spanish version of decimal and saying decimal with a Spanish pronunciation for the English version.)

This is called a decimal *(with English pronunciation)* point. Es un decimal *(with Spanish pronunciation)*. *(It's a decimal.)*

This interchange is confusing for several reasons. First, it is confusing because of the rapid fire codeswitching between Spanish and English that takes place. Second, the dot that the teacher wants his

students to learn is a decimal point, but it is never put with any numbers and, therefore, does not seem to gain any meaning other than being a dot. Third, the Spanish term for one-tenth is confused with the English term, "decimal," which sometimes is pronounced as a Spanish word and sometimes as an English word. At no time are the words written, and at no time is the difference between the terms explained.

Thus far, I have emphasized how little mathematics was spoken. Terms were used with little extended explanation about their meanings and numbers were used without their names being verbalized. It, however, is important to note that the mathematics lessons were not conducted completely in silence. In both classrooms, lessons were characterized by positive, active interactions between teacher and students and between students themselves as they worked together to solve problems. The issue is that, within these interactions, the language of mathematics was strikingly absent. When it was used, teachers and eventually students used it less than appropriately. Based on the misunderstandings some students expressed in the interviews and on the errors some students made in solving problems, it can be assumed that one result of the absence of clear mathematical talk is that, for them, fractions were simply two numbers that were written one *over* the other and decimals were numbers that had a dot somewhere between the digits.

Procedural and Decontextualized Explanations

The second pattern of discourse is related to the first and has to do with the nature of mathematical explanations. The teaching of mathematics in these two classrooms can be characterized as being procedural with little or no development of concepts. For our discussion, a procedural emphasis in teaching mathematics aims at establishing the steps that should be taken to solve a problem. It introduces a student to traditionally accepted algorithms or steps. Doing mathematics requires some knowledge of algorithms, but it also requires a good deal of conceptual understanding in order to know why and how the steps should be undertaken. In other words, it is not enough to know procedures. However, from the observations conducted, it can be assumed that the two teachers in this study understood the teaching of mathematics to be to provide students with prescribed procedures for performing a calculation.

As a result, much of what the teachers said was in the form of directions that students had to memorize. For example, one teacher after calling students' attention to the lesson, offered the following as an introduction: "When you add like fractions, you add numerators and put it over the denominator." There was little follow-up on what "like fractions" meant or what "it" referred to. Immediately after this, students began solving problems.

Figure 2 is taken from a sheet of directions students were given in order to do another day's lesson on simplifying fractions. A word problem is included that is intended to demonstrate why one would simplify a fraction. However, there is little connection between having a box of cans that is partially full ($16/24$) and the question that asks for the "lowest-term fraction," and no discussion is initiated by the teacher to make some sort of cognitive connection. The question of a simplified fraction is left appearing seemingly irrelevant to the rest of the situation regarding cans. That there is no discussion regarding the need to simplify the fraction is consistent with the belief that the objective is for students to learn the procedures for doing the simplification. In fact, the discussion during the lesson centered on the diagram that outlines the steps students should follow to solve the word problem at the beginning.

Figure 2 was not included in the electronic version of this publication.

The instructional emphasis on procedures also seem to encourage the use of words or phrases that carry little

mathematical meaning and that can be ambiguous. The following are some examples of what teachers said as part of their explanations.

- "Can you *go down* any *lower*?" (referring to further simplifying $16/10 = 8/5$)
- "Can you *fix it*?" referring to the above equivalent fractions.
- "You can *break 2/8 down* even more."
- "We did *plus* yesterday and today we'll do....(the teacher writes $3.6 - 2.7$ on the overhead and never finishes the sentence).

Unfortunately, there were few elaborations of the meanings of these statements nor were there any clarifications of ambiguous referents.

Another seeming consequence of an emphasis on procedures has to do with the use of few concrete items, pictures, or diagrams to reinforce the teacher's presentation. Both teachers always used an overhead machine but this was to display directions, examples of problems, and sets of problems students were to solve. One teacher liked to display what he called a "flow chart" (an example of this was presented earlier) that outlined an algorithm. Although this seemed to be very helpful to students in solving the day's problems, it was not sufficient to contribute to their understanding of what they were doing or why.

Furthermore, since there was little attention given to establishing the mathematics register as discussed previously, the teachers saw little need to write any more than the minimum words needed for a problem or for describing procedures. There were also few instances where the teachers pointed to what they had written on the overhead as they talked about it. Consequently, the instruction was highly decontextualized, forcing students to depend on their ability to listen and to make connections between what they heard and anything that was on display. In this situation, key mathematical words or phrases, such as the names for fractions, can easily be confused (e.g., two sixteenths for two sixteens). The following is an example of how the lack of making concrete what is being discussed can make the discussion incomprehensible. In this instance, the teacher has presented the procedure of using the cross products of two fractions to check whether the fractions are equivalent and is verbally explaining what is on the overhead transparency (Figure 3). The teacher's intention is to have the students write an equal sign (=) or a not equal sign (\neq) after cross-multiplying numerators and denominators.

Figure 3 was not included in the electronic version of this publication.

T: Remember cross products, productos cruzados (*crossed products*)? I want someone to come and do this problem. Be sure to write the symbol. Everybody understand? Write the symbol.

Seeming to understand the teacher's directions, the student who sits at the front of the class working the example draws a circle around the denominator and numerator of opposite fractions as shown in Part A at the bottom of Figure 3, and says this is the "symbol." The teacher responds that there is an error and suggests that the student try again to put the correct "symbol"; the student erases his first answer and writes the \neq but in a place that is not relevant to the intended mathematical statement (Part B at the bottom of Figure 3).

The foregoing demonstrates how easily mathematical discussions can become unclear when referents are ambiguous. In this case, there were many "symbols" to choose from and no steps were taken to make concrete and specific which symbol the teacher was talking about. The discussion would have been clearer if the teacher had pointed to the appropriate symbol on the transparency. The teacher also could have

pointed to the equal and unequal signs as he introduced the lesson and established what each one meant and how they were to be used during the lesson.

Little Spanish Is Used

The third pattern to emerge is that, in both classrooms, very little Spanish was actually used. The Spanish that was used could be classified into two categories. I call the first category "instrumental use." Both teachers, as they conducted their lessons, tended to use Spanish as an "instrument" to call attention to the lesson, to reinforce instructions, or to punctuate a statement. In these instances, Spanish usually consisted of single words or very short phrases such as "Andale" ("Hurry up") or "Este fraccion se llama mixto" ("This fraction is called mixed"). The second category can be thought of as "markers of solidarity." The teachers would use Spanish to give encouragement or to motivate the class; it was also used when the teacher worked individually with a student almost as a private but shared mode of expression. However, in these individual sessions, Spanish still was not used to explain any concepts.

Interestingly, both teachers felt strongly about the need to use the two languages in their teaching. The method they used, however, was a concurrent translation approach that made their speech very confusing. One teacher habitually codeswitched very rapidly between the two languages (e.g., "Who knows? ¿Quien sabe? Who knows?"). Again, the Spanish that was used was very limited as in this example from the other teacher:

We were talking about equivalent fractions, fracciones equivalentes. Who can tell me what an equivalent fraction is?

This same teacher frequently wrote the objective of the day's lesson on an overhead transparency in both English and Spanish (see Figure 2) and would read the English portion to the students but would sometimes omit reading the Spanish version or would read it in a haphazard manner so that the ending faded out and could not be heard.

As can be seen from the example in Figure 2, Spanish is used to set the algorithmic procedure for solving problems. It is not used to explain the mathematical concept, and even though much of the instructional talk in both classrooms is oriented toward learning procedures and not toward concept development, Spanish is still seldom used. Overall, in both of these classrooms, very few whole thoughts were conveyed in Spanish although for some students this was the only language of proficiency.

These two teachers represent an interesting issue that appeared to be common among the other teachers in the larger study. These teachers expressed a good deal of concern that their NEP students understand instruction and they had positive intentions for using both languages. However, they saw themselves as faced with teaching a group of students that to them was distinctly divided between Spanish-speakers and English-speakers. In their interviews, they indicated that they thought that the only method that was practical to use in this kind of situation was a concurrent translation approach even though they easily became fatigued switching back and forth. They had not thought about other possibilities for organizing instruction for the different languages. Add this to a difficulty in using the mathematics register in Spanish and it is easy to see why little Spanish was used.

Summary

The purpose of this paper was to initiate a much needed discussion on why language use should be studied as part of our attempts to better understand the factors that hinder or promote the learning of mathematics by students whose first language is not the dominant language of instruction. The focus of the study was on discourse characteristics of the teacher since the premise is that it is the teacher who is the primary model of what is to be learned, enculturator of the subject matter, and engineer of effective learning environments.

As can be seen from the foregoing results of a naturalistic look at how teachers use language to accomplish their education objectives in mathematics, there are many issues that emerge and that should be of concern. The first is how little mathematics was actually spoken. Although the classroom environment could be characterized as verbally active, the actual mathematics instruction could be characterized as almost silent. Secondly, when it was spoken, it was not always correct, appropriate, or unambiguous, and it was not related to the development of meanings. Thirdly, and most crucial given the nature of the student population, it was not always spoken in the only language of proficiency for some students. Furthermore, mathematics was spoken in a manner that required that students rely on their weakest ability in a second language, listening, and that put them at an additional disadvantage.

Language is such an inherent part of human activity that it is easy to take it for granted and to overlook its critical role in learning. In light of this role, talk in mathematics is not simply talk for talk's sake. Rather, "talking mathematics" is creating mathematical meanings through the use of language. Furthermore, students learn the language of mathematics from others (either teachers or students) who have mastered it and by actively using it. In these classrooms, in spite of all other positive instructional techniques and based on what has been presented, it should be asked: How is mathematical meaning and understanding, and thereby mathematical ability among students, being developed? From field and videotape observations of students' behaviors during the mathematics lessons and through interviews, it can be assumed that much of what they did was done by rote memory or by guessing.

Some Concluding Remarks

It should be pointed out that the teachers in this study effectively utilized many instructional strategies currently avocated by educators. Their education goals were very clear and positive for their students. As mentioned earlier, both teachers were unique in that they enjoyed teaching mathematics and made it a critical portion of their daily curriculum. There were many good things about their teaching and many good things they wanted to do but fell short on.

1. Both teachers were consistent about teaching mathematics in that the subject was taught everyday at the same time in the morning for a relatively lengthy period compared to other subjects.
1. Both teachers frequently attempted to relate the mathematics to concepts students seemed to already know (e.g., money and candy bars). However, the connections were not always clear, and therefore, references to these everyday examples appeared to be irrelevant.
2. Both teachers were systematic in setting norms for group work. Students understood how they were to work together and what they were to accomplish by doing so. However, the mathematics problems students were given were not appropriate for group work. The problems were ones that could be better done individually.
3. Both teachers had good rapport with their students and spent a lot of time working individually or in small groups with them. The students appeared to be eager and willing to participate in the mathematics lessons, and when they were reluctant, it was clear that they did not quite understand the

mathematics.

The teachers presented in this paper are not unlike most of the other teachers who were observed in the larger study (Khisty, McLeod, and Bertilson, 1990) nor are they atypical from the majority of teachers found in school districts around the country. Traditionally, mathematics instruction has been characterized by very little talk, vague use of words, and incomplete explanations (e.g., Pimm, 1987; Good, Grouws, and Ebmeier, 1983). In essence, therefore, these teachers are following current typical patterns of teaching mathematics. However, this is a period of major change in what is considered to be appropriate instructional strategies in mathematics. The National Council of Teachers of Mathematics (NCTM) has focused attention on the role of language in learning mathematics, and lists learning to communicate mathematically as one of its five major goals for mathematics students (NCTM, 1989).

In light of this new emphasis, a study of this nature provides new insights and raises new questions regarding the teaching and learning processes in mathematics with a student population that is relatively educationally high-risk and that historically has been underrepresented in mathematics and related professional areas. First, what are bilingual teachers' knowledge and abilities in explaining mathematics in their student's primary and secondary languages? Secondly, what are bilingual teachers' understandings about the mathematics register and what are their abilities to use and develop it effectively? What are strategies for developing the mathematics register in both languages? Lastly, a study of this nature raises larger policy issues (Khisty, 1991) including what is the role of teacher training in developing teachers' abilities to speak mathematically themselves and to develop students' abilities to speak mathematically in their primary language?

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