NCBE Program Information Guide Series, Number 15, Fall 1993

REFORMING MATHEMATICS INSTRUCTION FOR ESL LITERACY STUDENTS

Keith Buchanan; Mary Helman

Note: Every attempt has been made to maintain the integrity of the printed text. Due to constraints of the electronic environment, some figures and tables may have been reconstructed or omitted.

INTRODUCTION

Schools across the United States are challenged to meet the needs of an increasingly diverse population of students learning English as a second language (ESL). One measure of this diversity, the amount of students' past experience in formal education settings, has particular impact on instruction. The group of ESL students who have had limited or interrupted schooling in their first language-literacy students-can be overwhelmed by new experiences in ESL and content area courses. In mathematics classes, they must learn in a linguistically and culturally unfamiliar environment, constructing understanding without the background knowledge that their classmates employ to make assumptions and process new information. All too often, these circumstances lead to frustration for both literacy students and their math teachers.

Reform in mathematics instruction is necessary to assure that literacy students have access to math content from the beginning of their formal education careers. This reform calls for modifications in the curricula and in the delivery of instruction. By integrating math and language teaching, innovative courses can provide the experiences that bridge gaps in literacy students' math knowledge, expand their communicative competence in English, and ultimately prepare them for success in future math coursework.

The literacy students referred to in this monograph face a unique set of challenges not only in language learning but also in achieving the level of mathematical literacy advocated for all students by the National Council of Teachers of Mathematics (NCTM). Although these two usages of the term "literacy" may seem quite distinct, current research suggests that the most effective content area instructional programs for ESL literacy students are the ones that incorporate instruction for math literacy with language building activities (Garcia, 1991).

This guide is intended to help educators design literacy math curricula based upon instructional priorities of local school districts as well as the instructional standards of the National Council of Teachers of Mathematics. The courses modify the original sequence and emphases of the objectives to take into account the ages, diverse English literacy levels, and previous math backgrounds of students in each class.

Suggestions are also made for staff development for math teachers so that they may integrate language learning techniques into each lesson and employ extensive collaborative learning experiences to practice and demonstrate understanding of mathematics objectives.

Finally, the guide provides background for both math and ESL/bilingual personnel to encourage collaboration between the two fields in curriculum development and advocacy for literacy students. The sample lessons illustrate one way in which math and language instruction objectives may be integrated with specific teaching strategies.

CORRELATING LANGUAGE BUILDING WITH MATHEMATICS SKILL BUILDING

In response to the call for the reform of mathematics education in the United States, the National Council of Teachers of Mathematics established a Commission on Standards for School Mathematics in 1986. The purpose of the commission was to establish a set of standards that would ensure quality, indicate goals, and promote change. In 1989, NCTM published Curriculum and evaluation standards for school mathematics, which presented fifty-four standards among four divisions: grades K-4, 5-8, and 9-12, and evaluation.

In response to the current societal expectations that all students have the opportunity to become mathematically literate, be capable of extending their learning beyond the twelfth grade, have an equal opportunity to learn, and be capable of understanding issues in a technological society, the NCTM Standards established five general goals for mathematical literacy. These goals are articulated for all students:

1. that they learn to value mathematics

2. that they become confident in their ability to do mathematics

3. that they become mathematical problem solvers

4. that they learn to communicate mathematically and

5. that they learn to reason mathematically (NCTM, 1989).

The NCTM Standards clearly state that mathematics instruction must provide opportunities for all students to meet these goals and, therefore, become mathematically literate. These five goals have special implications for math teachers who are working with literacy students. While these students have had many experiences outside the school, most of these experiences have not always prepared them for success in formal classroom settings. They have had to solve problems, communicate, and reason, but in ways that are not generally found in math textbooks. Math teachers can make math meaningful for literacy students by designing instructional activities that build upon students' real life experiences. Learning math skills that directly relate to the jobs they report to after school each day, for example, will help students learn to value mathematics. Math lessons that provide challenging problem-solving activities at which students can succeed help to build their reasoning and problem solving skills, as well as their confidence. For students to learn to communicate mathematically, they need opportunities to hear math language and to speak and write mathematically.

Traditional notions of mathematical competence continue to change as expectations for workers continually rise to match the technological and problem solving demands of the work place. It is no longer sufficient for students to be able to perform the four basic operations (addition, subtraction, multiplication, and division) using algorithms. While teachers cannot neglect teaching these basic operations and algorithms, they must also provide instruction at an appropriate level that will allow each literacy student to move toward full realization of all five outlined goals of mathematical literacy.

NCTM STANDARDS AND EFFECTIVE INSTRUCTIONAL STRATEGIES FOR LITERACY STUDENTS

In 1991, NCTM produced a companion document to the curriculum standards. This document, Professional standards for teaching mathematics, provides guidelines for teachers designing an environment in which all students will develop mathematical literacy (NCTM, 1991). The guidelines require significant changes in classrooms for literacy students. Five of these changes follow, with examples for literacy classrooms.

1. Select mathematical tasks that engage students' interests and intellect.

Although the math concepts for literacy students may be at a primary level, the interests and intellectual abilities of these students are not. Selecting tasks that can bridge these wide discrepancies in ability levels is a challenge for math teachers. For example, in a lesson on calculating percentages, younger students might calculate the percentage of tax on a bicycle purchase, while older students use their pay stubs to calculate the percentages of various categories of withholding.

2. Orchestrate classroom discourse in ways that promote the investigation and growth of mathematical ideas.

Orchestrating discourse for literacy level ESL students in the classroom requires the teacher to attend to teaching English in the content area, which includes both the language specific to math and additional English language skills. For example, when teaching that an obtuse angle is greater than 90 degrees, the teacher will not only have to teach the vocabulary word "obtuse," but may also have to teach the use of the -er suffix to show comparison in the word "greater."

3. Use, and help students use, technology and other tools to pursue mathematical investigations.

Many literacy students are unfamiliar with the basic tools associated with mathematics such as rulers, protractors, calculators, and computers. Literacy students need opportunities to make optimum use of all the tools of mathematics. When working with estimation of lengths, for example, students can use both standard and metric measuring tools to find things they always have with them (e.g., wallet, dollar bill, thumb nail) that measure approximately one centimeter, one decimeter, one meter, one inch, one foot, or one yard. They can then use these items to estimate the length of objects within the classroom, check their estimates with the actual tools, and use calculators to find the percent of error in their estimations.

4. Seek, and help students seek, connections to previous and developing knowledge.

To make connections with students' prior experience teachers must become familiar with the backgrounds of literacy students. Working in collaboration with other content and ESL teachers in the school will help the math teacher provide connections with the knowledge students are developing in other classes. When students are studying data analysis and graph making, for example, the math teacher can collaborate with science and/or social studies teachers to build connections with work in other classes. For example, the students can graph data they have collected in other classes and analyze that data for measures of central tendency such as mean, median, and mode.

5. Guide individual, small-group, and whole-class work.

Literacy math students benefit from a variety of instructional settings in the classroom. The teacher must guide students through individual, small-group, and whole-class activities. For example, the introduction of a new set of vocabulary or manipulatives to the whole class can build listening and responding skills. Small-group work allows students to use language to talk about the math tasks at hand while they work to solve nonroutine problems. Individual work settings assure that all students can process lessons at their own rates of learning.

DESIGNING APPROPRIATE CURRICULA AND ASSESSMENT

A new literacy math curriculum should acknowledge questions such as:

- Who are our literacy students, and why are they unsuccessful in our present math courses?
- If an ESL student is assessed as three or more years behind his classmates, what exactly does that student need to learn?
- What is the most efficient way for students with limited amounts of time in school to learn what their

classmates already know?
- How can math teachers be expected to teach language?
- Why is it appropriate to separate literacy students from other math students for a time?
- How should literacy students' understanding of math be assessed?

Responses to the questions listed above should be used to guide curriculum development by coalitions of educators from both math and ESL/bilingual backgrounds who are knowledgeable about both the school district's math objectives and the needs of second language learners from various age groups. Math instructors judge the relative importance of existing instructional objectives and, along with ESL/bilingual personnel, develop specific teaching strategies.

CLUSTERING OBJECTIVES

Literacy math classes aim to teach a number of years of conventional math classes in a condensed period of time. In many cases, it is appropriate to cluster similar learning objectives across grade level boundaries. These clusters of objectives make the most efficient use of students' time in the literacy math class and also recognize that, often, older students do not require as much time to master objectives normally taught in younger grades. For example, after students have learned that "addition" means joining two or more sets, have developed an understanding of "place value," and have had experiences with regrouping, they are ready for addition. A typical mathematics program of study may contain the following objectives:

First Grade: Add sums to 10.

Second Grade: Add sums to 18.

Demonstrate with place value materials addition with regrouping for numbers less than 1,000.
Add two-digit numbers, regrouping as necessary.
Add three and four one-digit and two-digit numbers, regrouping as needed.

Third Grade: Use the addition algorithm to find sums to 1,000, regrouping as necessary.

All these objectives can be clustered in one objective stating, "Add two or more numbers using regrouping." In addition to saving class time, the clustering of these objectives reduces the artificiality of structuring lessons where, for example, students only solve problems that involve numbers less than 100 and do not require regrouping.

Clustering objectives also offers opportunities to integrate a variety of math strands into one lesson. In a geometry unit, for example, a group of students may estimate the cost of carpeting the classroom. The objective for the lesson would read, "Identify the space inside a plane shape as its area. Find the area of simple polygons." However, in order to carry out the activity the students will also demonstrate their understanding of the following objectives:

- measure lengths of objects using customary units
- multiply whole numbers, regrouping as necessary and
- multiply whole numbers by decimal numbers.

These math skills are being used by the students in a real life setting to solve a problem while mastering another objective. Thus, the teacher can assess mastery of the previously taught content and reteach where necessary while continuing to move through the curriculum.

THREE IMPORTANT VARIABLES: STUDENTS' AGES, DEVELOPMENTAL LEVELS, AND ENGLISH PROFICIENCY

The essential math objectives identified by local school jurisdictions should remain unchanged for literacy math students. Literacy math curricula, however, modify the sequence of the objectives by clustering and condensing them. Next, specialized teaching strategies are developed. All the strategies take into account students' ages, developmental levels, and English proficiency levels. Few published materials exist that teach early math skills with a range of age-appropriate approaches for beginning English learners.

Students' Ages

Innovative strategies need to be developed for seventeen year-old students with beginning English skills, as well as for fourth graders whose first experience in school occurs in American schools. Older students benefit particularly from math curricula that take into account their previous life experiences, such as problems involving money or their new school environment. For example, high school students who are studying ordinal numbers could be given practice identifying the periods of their school schedules or explaining the order of their lockers in the hallways.

The fourth grade math literacy student faces a smaller developmental gap with peers, yet may still need a period of specialized instruction. The texts and materials that native English speakers use to learn about ordinal numbers may not interest a student whose previous learning experience has never originated in books. Instruction with concrete experiences, especially incorporating math manipulatives, are effective bridges to formal math class education for literacy math students of all ages.

Students' English Proficiency

Students' English proficiency is a second consideration for developing appropriate instructional strategies. In the lesson on ordinal numbers, beginning proficiency students could complete an oral activity combining their understanding of colors with identification of the order of colored objects demonstrated by the teacher on an overhead projector. More advanced students could describe the exercise in writing. In general, less proficient learners depend more on the teacher or other students to model expected work and class behavior. A literacy math classroom will have a different look because it is enriched with extra attention to language. Charts with important vocabulary and language structures fill the walls, along with writing by the teacher and students.

Students' Developmental Levels

The third aspect of designing appropriate curricula is the wide range of students' developmental levels. Multiple learning strategies are necessary to reach both those students who show understanding of objectives after just a few activities and those who may need continued reinforcement. Literacy math teachers report that they are constantly revising curricular objectives to break them into smaller, simpler pieces, and revising directions to incorporate previously studied vocabulary and activities. Many teachers also modify their overall teaching plan by spiraling out of an objective before it has been mastered by many in their classes, then returning to it after a period of time spent working in another area. For example, after a week spent on a unit on Mental Math and Estimation, the teacher could redirect the class with individualized lessons on operations, incorporating the estimation skills students learned in order to predict their answers. When they return to the Estimation unit, the practical value of the lesson will be clear.

SUPPLEMENTING MATH TEACHERS' KNOWLEDGE

The actual curriculum publication will be especially useful for math teachers if specific teaching strategies are integrated throughout. When time and funding are limited for retraining math teachers to work with LEP students, specific teaching suggestions may provide a linguistic background or ways to simplify teacher talk. For example, in the ordinals lesson, ESL/bilingual personnel could add notes in a teachers' guide concerning the difficulty with oral production of -th endings on ordinal numbers. An appropriate learning objective for the math class could focus on the aural discrimination of similar-sounding numbers, such as "thirteenth" and

"thirtieth," while a cooperating ESL/bilingual teacher could provided further pronunciation practice during language classes. (More examples of incorporating language, math, and teaching strategies are provided in specific examples at the back of this guide).

ASSESSING LITERACY MATH STUDENTS' PROGRESS

Just as mathematics content and instruction change to meet the needs of literacy students, teachers need to find different ways to describe and assess literacy students' learning in mathematics.

> As instruction for language minority students has moved in the direction of teaching language and content-area skills in context, assessment has begun to incorporate a wider variety of measures that more closely reflect the types of tasks that students are asked to perform in classrooms or in real-life settings. In this sense, school-based assessment procedures are becoming more authentic. Changes in assessment are especially important for language minority students and those acquiring English as their second language. (Pierce and O'Malley, 1992)

The function of these assessment methods is to measure growth or progress toward meeting established goals and standards. However, the point from which this growth is measured varies greatly from one literacy student to another but is usually far below the math and English levels of the peer group as well as the math and reading levels of first language peers. It is from this point that growth in both mathematics skills and English language skills must be measured. Reliance on paper and pencil tests is particularly inappropriate for literacy students. Decoding the language of a test may often hinder students rather than actually allowing them to demonstrate what they understand in math. The use of a wide variety of assessment methods will provide a more complete picture of each literacy student's progress.

The NCTM Curriculum and Evaluation Standards (1989) propose changes in the methods and processes of student assessment. The Evaluation Standards propose that:

- student assessment be an integral part of instruction
- multiple assessment methods be used and
- all aspects of mathematical knowledge and its connections be assessed.

The first proposal is that student assessment be an integral part of instruction. As delivery of instruction is modified for literacy math students, assessment of these approaches must be similarly reformed. Ongoing assessment in the classroom allows teachers to revise instruction as needed. Options include:

- **Spot Checks**: As students work with manipulatives or in solving problems, the teacher can circulate and look over shoulders to see how the students are doing. The students can also demonstrate understanding by using "thumbs up/thumbs down" to indicate which of two answers is correct, holding up a manipulative such as the correct fraction bar, or writing their estimate on a scrap of paper and holding it up.

- **Checklists**: The teacher can use a checklist to assess seatwork or classwork. A "+" can be used to indicate that a desired behavior has been observed. By using a checklist weekly and retaining the copies, growth can be observed over time. During a class period, the teacher can use the entire checklist to assess a few students or only one item, such as "explains mathematical process using appropriate vocabulary," to assess all students.

- **Anecdotal Records**: The teacher can use an anecdotal record sheet to record observations during or immediately following a lesson, to record information about a student over time, and to determine patterns of development or areas of need. Dated short notes or phrases can be recorded on individual student sheets and filed.

The second NCTM proposal involves using multiple assessment methods. Instead of focusing on what students do not know, we need to focus on what they do know and use that information to help them progress. While grades from tests and quizzes have a legitimate place in assessment, they comprise only one part of the total picture of a student's math knowledge. The use of a rating scale, or rubric, and the compilation of a portfolio are two other methods of assessment which give a broader view of what the student knows.

- **Rating Scales/Rubrics**: A rating scale, or rubric, is used to score a paper or problem holistically. "The rubric should be created to reflect the specific important elements of each open-ended question" (NCTM, 1991). Students receive scores between zero and the highest score as the reader looks at the problem as a whole. A student who made no attempt would receive a score of zero, whereas a student who successfully completed all the important elements required by the question would receive the highest score, perhaps a five or six, depending on the scale chosen. The rubric assigns points based on various aspects of the response such as the student's understanding of the problem, the steps used toward solution, and the completeness of the answer. A second rubric could be used to score the same piece of student work for language usage.

- **Portfolios**: A portfolio serves as the assembly point for samples of student work collected in a variety of settings and through a variety of methods. The teacher may establish a set of items to be placed in the portfolio, and the student may select others. These could include problem-solving activities, long-term assignments, a math log, writing samples, best homework paper each quarter, worst test along with a short writing piece telling what the student learned from the test, and other items which indicate the breadth of the student's math knowledge.

The third proposal from the Evaluation Standards is that all aspects of mathematical knowledge and its connections be assessed. According to Short (1992), assessment should be viewed holistically in an integrated language and math course, but it is sometimes appropriate to separate language issues from math concepts. Because understanding the language of a word problem requires a different set of skills than solving the same problem, we must find ways to assess both separately. All the assessment methods listed above can be used to assess language development (e.g., writing samples in the portfolio or a checklist item about using the correct English vocabulary). Methods for holistically assessing the four areas of language development include:

- **Listening.** Anecdotal records can be used to record progress over time by responding to questions such as:

  Can the student attend to teacher directions? Can the student discriminate between similarly sounding target vocabulary words? Does the student listen and respond appropriately in cooperative group settings?

- **Oral.** A checklist could include items such as:

  The student responds to teacher questions. The student uses questioning skills. The student uses English to discuss math. The student uses the vocabulary from the lesson appropriately. The student explains his/her reasoning orally to a partner, the class, or the teacher.

- **Reading.** When assessing homework/classwork, the teacher can ask:

  Does the student recognize vocabulary words from a list? Does the student follow simple written directions? Does the student identify what a word problem is asking?

- **Writing.** A writing sample can ask the student to:

  Copy vocabulary into a student notebook. Define a vocabulary word. Write a story problem

following a model. Give a written explanation of a process.

ADDING TO MATH TEACHER EXPERTISE WITH ESL TECHNIQUES: STAFF DEVELOPMENT NEEDS

A recent questionnaire of 300 secondary school math teachers polled their confidence in meeting the needs of literacy students in conventional math programs (O'Malley, 1992). At 43 schools with a wide range of language minority populations, math teachers were asked to agree or disagree with the statement, "I can successfully work with ESL students who are judged to be three years behind their peers in math background" (based on a comparison with the local school district's grade-level math objectives). Fewer than 25 percent of the math teachers responded that they could be successful in instructing these students. This self- reported lack of confidence helped to define the parameters of the math literacy student population and led to the establishment of special math literacy classes and training for math teachers in the school district.

The polled teachers were employed in schools that had experienced rapid growth in language minority populations, including large proportions of literacy students. When these students enrolled in math class, both teachers and students were frequently frustrated. Like other mathematics teachers in many areas of the country, these teachers were concerned about their lack of training to meet the needs of LEP students in their classes, whether in sheltered courses or in classes with a mixture of native English and nonnative English speakers.

In some schools, ESL teachers have taken over all content area instruction for literacy students, yet the advantages of mathematics teachers assuming responsibility for the math education of LEP students are clear. Experienced math teachers have a repertoire of math instructional strategies from which they may offer students a variety of ways to approach an instructional objective. Math-certified teachers are familiar with the progression of math curricula and are able to provide the kinds of math experiences that lead from the literacy math class to subsequent ones. In many states, high school students are granted credit toward graduation only when the teacher of a sheltered content course is certified in that content area. Therefore, when ESL teachers instruct a sheltered math course, students will receive credit for ESL, rather than the math credits required for graduation.

Math teachers working with literacy students, however, need training that adds to their field of expertise in order to make math accessible to language minority students. Hamayan's (1990) description of six roles for content area teachers of language minority students can be extended with examples of teaching strategies particularly appropriate in literacy math classes, as follows.

1. Teachers become mediators and facilitators for content learning by presenting material that not only meet local school district objectives and NCTM standards, but also take into account the specific gaps in literacy students' math knowledge. They modify the sequence of learning objectives for each group. They tie new knowledge to previously acquired skills. For students with little formal education background, they model persistence in problem solving activities.

2. Teachers are facilitators of acquisition of English through daily emphasis on the specific vocabulary and language structures of each lesson. Wall charts display vocabulary in lists and in context. Teachers use demonstrations and math manipulatives in presentations. They take into account oral, aural, written, and reading aspects of second language learning and continually assess students' understanding of both math and language. Teachers model how to keep a math notebook so that literacy students can track their own progress.

3. Teachers serve as language models by modifying oral presentation methods through rephrasing and emphasizing key elements of daily lessons. They model their own writing and problem solving especially by

using overhead projectors. By writing what they say, they allow students to see them think aloud and make errors.

4. Teachers act as mediators of mainstream culture for literacy students. Teachers model and reinforce appropriate behavior for formal classroom settings. They show interest in students' native cultures and emphasize those aspects that students share in an ethnically and linguistically diverse classroom. They may create math experiences around special events, holidays, and school traditions.

5. Teachers become advocates for literacy students. They establish a classroom atmosphere that is accepting, decreases competitive activities, and values participation from every student. They seek to identify and show value for students' prior experiences and build math experiences around them.

6. Teachers collaborate with administrators and other teachers to assist students who may need remediation, to seek assistance from ESL/bilingual teachers to set appropriate education goals for literacy students, and to make social services available to students with needs beyond the math class.

Teachers of literacy math students need extensive and varied opportunities to develop expertise in these roles. Math teachers can add to their existing skills through course work, assistance from curriculum guides, and most importantly, through collaboration with ESL/bilingual staff and peer support models of staff development. (Milk, Mercado, and Sapiens, 1992).

SAMPLE LESSONS

The following lessons are samples from a curriculum for middle and high school math literacy classes in Fairfax County, Virginia. (Helman and Buchanan, 1992). Although the objectives in these sample lessons are commonly found in math curricula around the country, they are not meant to prescribe specific learning objectives for other literacy math projects, which should be based on instructional priorities of individual school districts. Rather, these samples represent one possible way language and math instruction can be integrated for a diverse population of literacy math students.

The curriculum follows the local mathematics program of study, but provides special strategies for literacy math students who have been assessed as being three or more grade levels below their peers in mathematics skills. The curriculum is intended to be a complete resource for teachers, providing all necessary practice and activities. These four sample lessons exemplify the diversity of activities and the scope of the objectives found in the literacy mathematics course.

Because the LEP population of the school district includes students who are native speakers of 57 different languages, literacy math classes are taught in English rather than through bilingual approaches. Math-certified teachers serve as the instructors; they are joined in some schools by collaborating ESL teachers.

New literacy math teachers participate in in-service training throughout the year, but information critical to teaching literacy students is also imbedded in each lesson. For each objective, teachers find a vocabulary list and a list of the materials they need to gather for a lesson, including math manipulatives. At the core of each objective are sections on language foundations and a mathematics component. The language foundation supplies background on language structures relevant to teaching the objective, while the mathematics component suggests specific instructional activities, emphasizing cooperative, concrete experiences.

NUMBER CONCEPTS AND THEORY

Objective 2: Count, read, represent with place value materials, group, and regroup numbers to 100.

Vocabulary: twenty, thirty, forty, fifty, sixty, seventy, eighty, ninety, one hundred, group, regroup, eleven,

twelve, thirteen, fourteen, fifteen, sixteen, seventeen, eighteen, nineteen, unit, rod

Materials: Number cards 1-9, Number cards 1 ten to 9 tens, 100s board, Overhead, Overhead pens, Transparency of 100s board, Base 10 blocks, Overhead base 10 blocks, Chart of numbers 11 to 20, Place value boards, Place value transparency, Blank transparency

Language Foundation

1. The new vocabulary for this unit will be introduced as part of the lesson. A daily review of the numbers and number words needs to be included in each lesson. Spend the first five minutes of the class reviewing vocabulary and material from the previous lesson(s).

2. For ESL students, the numbers 13 and 30 are "close confusers," as are the other pairs such as 14 and 40, 15 and 50, etc. At the beginning, students simply have great difficulty hearing the difference in the two words and making the distinction. Take great pains to pronounce them distinctly and write them as you say them to help students build the ability to hear the difference. This is a listening skill that may take more than a year to develop.

3. Throughout the year, as you work with the base 10 blocks, you will need to decide if you will call the small cube a "one" or a "unit." Either is fine, but at the beginning be consistent when you name it for the students and when you refer to it. The same applies for the "rod" or "ten" and the "hundred" or "flat."

4. Following the Mathematics Component you will find a series of activities that can be used to reinforce both the vocabulary and concepts introduced in this lesson. These can be used for the next few days as you assess the mastery of this objective. They can also be used as daily review activities in the future.

5. Encourage students to talk to one another as they work in pairs on these activities. Math language must be used in order for it to be built. Encourage the use of English, but do not discourage students from using their first language(s).

Mathematics Component

Place the number cards 1 to 9 in random order on the chalkboard tray. Have a student put them in the correct order and count them aloud. Then place the cards for 1 ten through 9 tens on the chalkboard tray and have a student place them in the correct order. Write the numbers on the chalkboard for each of these above the corresponding card (e.g., above the card reading "7 tens," write "70"). Introduce the word names for each card, writing these above the number. Pass out 100s boards and put the transparency of the 100s board on the overhead. Have the students count the first row. Then help them locate the tens column. Count by tens with them. Give each pair of students 20 unit cubes and four rods. Using the overhead base 10 blocks, model for them that 10 units equal one rod. Have students do the same, counting the units cubes. Place a rod on each of the tens from 10 through 100 on the transparency. Count by tens. Have the students count by tens. Add the rest of the number line to 100, circling the tens as you indicate them and name them.

Have students place a unit cube on each square from one to 11 as you model doing it on the transparency. Count each cube as you place it. As they place one cube on the 11, emphasize the number 11. State that it is one ten and one more unit. Put up the chart of numbers from 11 to 20, indicating that 11 is the first number on this chart. Introduce the rest of the numbers up to 20, indicating for each one that it is one ten and ____ more (e.g., 17 is one ten and seven more). Write a number between ten and 20 on the overhead, saying it as you write it. Have the student count out that many cubes. Repeat. Conclude the lesson by placing one cube at a time on the 100s board transparency as the class counts aloud from one to 20. Repeat placing rods on the tens column and having the class count by tens to 100.

Pass out a place value board to each pair of students. Show them the unit cube at the top of the ones column

and the rod at the top of the tens column. Place the place value transparency on the overhead. Explain that only ones can go in the ones column and only tens can go in the tens column. Put a pile of 36 cubes (use the wooden ones) on the overhead in the ones column. Begin by counting them one by one, placing them end-to-end to form a rod. When you have ten, lay a rod beside them to show that it is the same. Tell the students you are going to trade one rod for the 10 units. Remove those ten cubes and place the rod in the tens column. Repeat for another set of 10 units, replacing them with another rod. Repeat until there are three sets of ten and six units left. Ask how many tens there are. Elicit the answer, "three." Count by tens to thirty and then continue counting the units-thirty-one, thirty-two... thirty-six. Write "3" in the tens column and "6" in the ones column. Say that three tens and six ones are thirty-six. Repeat with 43 cubes.

Give each pair of students 25 cubes. Work with them to get two tens and five ones, counting to get twenty-five. Have two pairs of students combine their cubes and repeat to get five tens and zero ones, counting by tens to get 50. Put six tens and 14 ones on the overhead. Ask the students to tell you or show you what needs to be done. Elicit the response that ten ones need to be traded for a ten, leaving seven tens and four ones. Count to get 74.

Write the number 62 on the overhead. Ask how many tens are needed. Elicit the answer "six." Ask the students how many ones are needed. Elicit the response "two." Place six rods and two units on the overhead. Have the students count with you to get 62. Repeat with other numbers between one and 100. Use other activities from the next page as needed.

Hundreds Board

1	2	3	4	5	6	7	8	9	0
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

ADDITIONAL ACTIVITIES FOR NUMBER CONCEPTS AND THEORY (NCT), OBJECTIVE 2

1. *Missing Numbers on a Number Line*: Using 5 x 8 cards, cover numbers on the number line. Let the students take turns telling you the missing numbers. For example, you might cover 4, 8, 11, 13, 14, 17, 19, 20, and so on.

2. *Missing Numbers on the 100s Board*: Using a blank 100s board transparency, fill in approximately half of the numbers. Let students take turns writing in the missing numbers.

3. *Using the blank side of the 100s Board*: Have the students write the numbers from 1 to 100 on their hundreds board using crayons. If they need help, tell them to turn the board over to find the numbers they need.

4. *Estimating and Counting:*

a. Give each group of four students a 100s board and a set of things to count. The groups may count unit cubes, two-colored counters, multilink cubes, green triangles from the pattern blocks, and so on.

b. Take a handful of multilink cubes. Ask students how many they think you have in your hand. Place the cubes on the overhead 100s board to count the actual number. See who came the closest. Repeat.

c. Have the students take turns getting a handful of whatever their group is counting. The others in the group should estimate how many were taken. The first student then lays the objects on the 100s board, counting to see whose estimate was closest.

5. *Geoboard Counting*:

a. Give each group of four students two geoboards and 30 to 40 multilink cubes. Have the students place their geoboards side by side to form a rectangle measuring 10 pegs horizontally by five pegs vertically.

b. Ask the students to count out 10 cubes and place them on the pegs along the top of the board. Have them continue to place their cubes until all are placed. Ask them how many tens they have and how many ones.

c. Ask them to turn their boards around carefully so the empty pegs are on the top. (You will probably need to explain the word "empty.") Ask them how many empty tens they have and how many empty ones. Ask them what number this is. Have them count the empty pegs to see if they get the same number.

d. Repeat by switching a few blocks in each group.

OPERATIONS

Subtraction Objectives

Vocabulary: take away, minus, left, subtract, difference, compare, count, back, trade, regroup

Materials:

Overhead

Objective 1: Two-colored counters, Cubes,

Objective 2: -, = cards ; Cubes, Ten Frame (one per student), Diffy Puzzle Transparency, Diffy Puzzle Game Sheet (one per student).

Objective 4: Base 10 blocks, Overhead base 10 blocks, Place value boards, Overhead place value board, Number cubes, Two-colored counters

Objective 5: Base 10 blocks, Overhead base 10 blocks, Decimal place value board

Objective 7: Two-colored counters, Cubes, Counting boards, Base 10 blocks, Place value boards, Paper

Language Foundation

1. The "-" (minus) sign should be seen as the sign placed between the numbers to be subtracted. We subtract numbers and separate sets. With manipulatives, the "-" sign stands between the parts. The sign tells us to subtract and is always placed between the numbers to be subtracted.

2. Subtraction has several interpretations. Two of the most common are "take away" and "comparison."

Take Away

Jesse has 6 pencils.
He gives 2 to a friend.
How many does he have left?

Comparison

Sana has 6 books.
Duane has 2 books.
How many more books does Sana have than Duane?

Both would be solved by the equation, "6 - 2 = 4." However, they represent two very different situations. The "take away" example is the one the students are most familiar with and demonstrates a given set of six objects from which two are removed. The "comparison" example is solved by placing a set of six and a set of two objects on a table and comparing the two sets by one-to-one correspondence. Students need opportunities to solve both kinds of problems.

3. For students who have difficulty regrouping across zero, provide exercises like these:

> a. For 108 we can think 10 tens and eight ones. We can also think nine tens and 18 ones. b. For 406 we can think of 40 tens and six ones. We can also think 39 tens and 16 ones. c. For 3004 we can think 300 tens and four ones. We can also think 299 tens and 14 ones.

4. For students who have difficulty adding or subtracting decimals in which different numbers of decimal places are used, play money-dimes and pennies-can be used to show that values such as 0.4 and 0.40 are equivalent. Count dimes and write as tenths (1 dime = 0.1, 2 dimes = 0.2, and so on). Now discuss the same amount with pennies (1 dime = 10 pennies = 0.10, 2 dimes = 20 pennies = 0.20, and so on).

Introduction to Subtraction

The intent of these lessons is to establish an understanding of subtraction and to relate written algorithms or rules to the process. Students need many opportunities to carry out the process using concrete materials. They should then move to recording their work, relating it to what they are doing with the manipulatives. Finally they will use the algorithm alone.

Trading in subtraction is closely related to trading in addition. However, more students have difficulty with trading in subtraction. Although trading in addition is closely related to place value work with 10 ones being traded for a ten, in subtraction, the trading of one ten for 10 ones (the opposite of trading in addition) is new.

In this unit, the objectives are listed, followed by directions for using manipulatives to teach the concepts. Specific lesson plans are not written for these lessons as instruction will tend to be individualized or provided to small groups of students.

Subtraction Objectives

1. Demonstrate an understanding of both forms of subtraction:

a. The comparison of two groups. b. The separation of one group from another.

2. Subtract with minuends to 20. Know that the order in which numbers are subtracted does change the answer. (Subtraction is not commutative.)

3. Write and solve subtraction sentences in both horizontal and vertical forms.

4. Subtract a two-digit number from a larger two-digit number, regrouping as necessary.

5. Subtract decimal numbers, regrouping as necessary. Numbers should contain no more than four digits.

6. Solve simple subtraction word problems. Write an addition or subtraction equation to describe a picture or group of objects.

7. Solve addition and subtraction equations with missing terms, working with sums up to 100 (e.g., 34 + _____ = 72).

DATA ANALYSIS, STATISTICS, AND PROBABILITY

Objective 2: Gather, organize, and interpret data using tallies and bar graphs

Vocabulary: statistics, graph, tally marks, tallies, bar graph, data, comparison, more, most, less, least, fewer, title, coin, penny, nickel, dime, quarter, altogether, vertical, up and down, horizontal, across, create

Materials: After School Activity transparency, Overhead pens, Crayons or colored pens or pencils, Pattern blocks, Blank paper, Plastic coins, Coin worksheet (one per student)

(Note: The "After School Activity Worksheet" and "Coin Bar Graph Worksheet" have not been included in the electronic version of this document.)

Language Foundation

1. **Unit**: Statistics

Lesson: Tallies and Bar Graphs

Vocabulary: *Tally* = mark used to count things; *Bar graph* = math picture used to show data or information.

2. The new vocabulary for this unit will be introduced as part of the lesson. Review previously introduced vocabulary including the names of the coins and the blocks.

3. A graph is a picture of information. We have already learned what a pictograph is. Who can tell the class what a pictograph is? Today we are going to learn about tallies and bar graphs. Tally marks are used when counting objects. One mark equals one item that you are counting. Bar graphs are another way to show information.

Mathematics Component

Put the After School Activity transparency on the overhead. Discuss the choices with the class. Ask students to come up to the overhead and make tally marks for their choices of an After School Activity. Students will need to remember their choices. Be sure to discuss the diagonal tally mark you make when you reach five marks. (If no item receives five marks, be sure to discuss the diagonal mark anyway.) Calculate the total values for each choice with the students. Explain to the students that the class will now complete the bar

graph using the tally totals. If you choose, you may have each student color in a block on the overhead bar graph to illustrate their earlier choice or you may do it yourself. It is important to ask questions about the results of the graph, as in the previous lesson. You will want to write your own questions at the bottom of the graph. For example, "How many more people chose sports than reading?" or "How many fewer votes for television than music are there?"

For the next activity, the students will work in groups. Distribute a mixture of pattern blocks to each group. Have each group sort their blocks by shape/color. Stack the sorted blocks next to each other to make a three-dimensional bar graph. Have each group discuss its bar graph and make up five questions about the graph on a blank sheet of paper. The groups will rotate around the room (to one of the other locations) and answer the five questions based on the bar graphs at their new locations. While the students are still at the new locations, they will present their answers and explain to the class how they decided on them. Let each student in the group read and answer a question. If appropriate, you may want to discuss why a question is relevant or not relevant. Collect the pattern blocks.

For the next activity, the students will continue to work in groups. Distribute a handful of plastic coins (do not include half dollar coins) and the Coin Worksheet to each group. Discuss the different coins and write the names of the coins on the board/overhead. Have the students copy the names of the coins in the appropriate places on their worksheets and complete the tally marks, totals, and bar graphs. Discuss with the students where the names of the coins and the numbers go on the graph. This is a horizontal graph so you may want to discuss how it differs from the last two graphs. Each group will then write five questions pertaining to its graph on the bottom of its own worksheet. Be sure that at least three of the questions are comparison questions (e.g., How many more quarters are there than pennies?).

PROBLEM SOLVING

Objective 4: Use "guess and check" to solve routine and non- routine problems.

Vocabulary: guess, check, closer, possible, reasonable

Materials: Overhead, Transparency Ticket Sales, Copy of Ticket Sales (one per student), Transparency of Marble Game, Copy of Marble Game (one per student), Transparency of The Garden, Copy of The Garden (one per student)

Optional: Paper bag with an apple or another object in it, Number Line

Language Foundation

1. "Guess and check" is a popular strategy used by many students in solving all kinds of problems, especially when they are not sure what to do. However, it is a strategy that is very helpful in special kinds of problems and needs to be taught. When a problem has so much data that making a list is a major task, guess and check is a useful strategy to start with. It is also a useful strategy when a student is asked to find only one of many possible solutions to a problem. When students use this strategy, they guess an answer and then check to see if it is correct. If the first guess was incorrect, they make another guess, repeating the process and gradually coming closer to the correct answer by making more reasonable guesses.

2. Helping the students understand the term "guess" is difficult. You might try explaining that it means "to think of an answer that could be right." A concrete way of presenting the term would be to use a paper bag with an object in it such as an apple and having the students guess what the object is. To help them make more reasonable guesses, give them a clue after each wrong guess.

3. You can help the students understand the idea of reasonable guesses and getting closer to the answer by

playing the game "Guess My Number." Use a number line on the board or overhead to mark their guesses. As the students guess, tell them "too high" or "too low" after each guess. Use the number line to help them get closer to the number. Explain that a reasonable guess is one that gets closer to the right answer. (You may need to explain the meaning of "too high" and "too low.")

Mathematics Component

Tell the students that they will use the five-step problem solving process (question, data, choose a strategy, solve, evaluate) using another strategy. Review the word "strategy" and the two strategies they have already learned-"act out or use objects" and "draw a picture." Tell them that they will use the strategy "guess and check" and show it to them on the chart. Use the activities in the Language Foundation to help them understand the meaning of the terms "guess," "check," "closer," and "reasonable."

Put the transparency of the Ticket Sales problem on the overhead and give each student a copy. Have them guess the answer to the problem. Tell them to add the two numbers together to check their guesses. Repeat as necessary, explaining that they should keep guessing until they arrive at the correct answer and that they should check their answers after each guess.

Put the transparency of the Marble Game problem on the overhead and give each student a copy. Have the students help you guess which four holes that might add up to 25 points. Tell them this is a "guess." Tell them to check their guesses by adding up the points for the four holes to see if they equal 25. Continue the process by helping the students make reasonable guesses until they arrive at a correct answer. Ask them to try to see if they can find another set of four holes that equal 25 points. Explain that sometimes there is more than one correct answer. Tell them that many times a guess and check problem will have more than one correct answer.

Put the Garden transparency on the overhead and give each student a copy. Explain that sometimes it is wise to make a table of the guesses so as not to repeat them. Explain how the table works. Have the class make one or two guesses to get started. Have the students finish the problem in small groups or pairs. Allow the students to use calculators. Again, the issue is using the strategy correctly. The computation should not get in the way. Assist those students who still need additional help with making reasonable guesses.

The following pages contain additional problems for practicing the guess and check strategy. Any of the problems can be used by the whole class if additional instruction on the strategy is needed.

Ticket Sales

Sara sold 12 tickets on Monday, 10 tickets on Tuesday, 8 tickets on Thursday, and 11 tickets on Friday. On which two days did she sell a total of 21 tickets?

Marble Game

Soo Won is playing a marble game. She shoots 4 marbles on the board. Each time a marble goes in a hole, she scores the number of points beside the hole. Soo Won scores 25 points for her 4 marbles. Which holes could the marbles have rolled into?

The Garden

Tomas has a garden with tomato and melon plants. Every tomato plant has 6 tomatoes on it. Every melon plant has 3 melons on it. Tomas has 222 tomatoes and melons in all. How many tomato plants and how many melon plants does he have?

Tomato Plants	Tomatoes	Melon Plants	Melons	Total
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.

Guess and Check Practice Problems

1. Deena and Linh are buying a pizza. They pay $15.00. Deena pays $3.00 more than Linh. How much money does each girl pay?

2. Ana has copper coins and silver coins in her collection. She has 59 coins. She has 15 more copper coins than silver. How many coins of each kind does she have?

3. Paints = $2.95
Stapler = $7.99
Ruler = $1.05
Scissors = $3.95
Glue = $1.05
Clay = $5.70
Calculator = $8.89
Juan buys two things that cost $9.04. What did he buy?

4. Alex and Marco walked 10.5 miles. Alex walked 2 times as far as Marco. How far did Alex walk? How far did Marco walk?

5. In the big fish tank there are blue fish and gold fish. There are 120 more gold fish than blue fish. There are 370 fish in the tank. How many fish of each kind are in the tank?

6. Alberta saves pennies and nickels. She has $2.05. She has 81 coins in all. How many pennies and nickels does she have?

7. There are 8 cows and chickens in the field. There are 20 feet. How many cows and how many chickens are

in the field?

8. Tickets are $2 and $3. Leah sells 7 tickets for $18. How many of each did she sell?

9. Chena, Nina, and Maria are in 3 lines. There are 3 more people in front of Chena than there are in front of Nina. There are two times as many people in front of Maria as there are in front of Nina. The total number of people in front of the girls is 11. How many people are in front of each of the girls?

10. Ahmed is counting cars in front of the school. He counts 8 Toyotas and 7 Nissans for every 9 Fords. 216 cars are in front of the school. How many are Toyotas, how many are Nissans, and how many are Fords?

Answers:

1. $6 Linh, $9 Deena
2. 22 silver, 37 copper
3. Stapler $7.99, ruler $1.05; or stapler $7.99, glue $1.05
4. Alex 7 miles, Marco 3.5 miles
5. 245 gold, 125 blue
6. 50 pennies, 31 nickels
7. 2 cows, 6 chickens
8. 3 $2 tickets, 4 $3 tickets
9. Chena 5, Nina 2, Maria 4
10. 72 Toyotas, 63 Nissans, 81 Fords

REFERENCES

National Council of Teachers of Mathematics. (1989). Curriculum and evaluation standards for school mathematics. Reston: Author.

Garcia, E. (1991). Education of linguistically and culturally diverse students: Effective instructional practices. Santa Cruz: National Center for Research on Cultural Diversity and Second Language Learning.

Helman, M., and Buchanan, K. (1992). Resource notebook for Focus on Achieving Standards of Teaching Mathematics (FAST Math). Fairfax: Fairfax County Public Schools.

Milk, R., Mercado, C., and Sapiens, A. (1992). Rethinking the education of teachers of language minority children: Developing reflective teachers for changing schools. Washington, D.C.: National Clearinghouse for Bilingual Education.

National Council of Teachers of Mathematics. (1991). Professional standards for teaching mathematics. Reston: Author. O'Malley, J. M. (1992). Project evaluation year 1, Project FAST Math. Fairfax: Fairfax County Public Schools.

Short, D. (1992). Integrated language and content assessment. What and how. Washington, D.C.: Center for Applied Linguistics.

Stenmark, J. K., (Ed.). (1991). Mathematics assessment: Myths, models, good questions and practical suggestions. Reston: National Council of Teachers of Mathematics.

Pierce, L. V., and O'Malley, J. M. (1992). Performance and portfolio assessment for language minority students. Washington, D.C.: National Clearinghouse for Bilingual Education.

The National Clearinghouse for Bilingual Education (NCBE) is funded by the U.S. Department of Education's Office of Bilingual Education and Minority Languages Affairs (OBEMLA) and is operated under contract No. T292008001 by The George Washington University, School of Education and Human Development, Center for Policy Studies. The contents of this publication do not necessarily reflect the views or policies of the Department of Education, nor does the mention of trade names, commercial products, or organizations imply endorsement by the U.S. Government. This material is located in the public domain and is freely reproducible. NCBE requests that proper credit be given in the event of reproduction.

go to HOME PAGE
http://www.ncela.gwu.edu